Reflections of a Youngish Mathematician

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So You Want To Teach Mathematics

Teaching ain't easy. When I first started teaching, it was as an undergraduate in Electrical Engineering. At my undergraduate alma mater there was a work-study position called "Peer Learning Assistant" (PLA) available to undergraduates. I was lucky enough to have gotten two positions as a PLA, one within the Electrical Engineering department and one within the Mathematics department. Part of the job duties were to take care of the "grunt" work (as we described it then) for the course we were assigned to. These were things like grading homework and quizzes, proctoring exams, making photocopies, etc. The other part of the job was to run "recitation sessions". These sessions varied based on what the professor wanted. For some courses, the recitation sessions were homework help and exam review, for other courses, the sessions were genuine teaching sessions where we had to go over a topic the professor didn't have time for in lecture, and still for other courses they were to help graduate students to run course labs (Engineering mostly). The PLA was in charge of and present at these sessions. The professor was never a part of them.

What was truly rewarding about being a PLA was that I was in a position of looking absolutely stupid. This was rewarding because it forced me to make sure that I knew what I was talking about. It was, or maybe I imagined it was, fairly easy to get "fired" from the position on account of incompetence. The level of respect that I could command in a classroom was far less than what a professor would automatically have by virtue of his/her position. I was probably no more than a year or two older than the students I was supposed to help.

While a professor could probably get away with an attitude of benign neglect or just outright callous indifference, a PLA could not. We were there to be the additional out-of-class support for students. So given that these were students I would see on campus for several years and that my job performance was tied to their general level of content. I really had to know my stuff and I really had to be able to explain it. Fortunately for me, I did know my stuff, or at the very least, made sure to know it by the time of the recitation session.

After finishing my Bachelor's degree, I went and worked in industry for a few years (between graduating and finding a full-time job, I did some part-time work (well, I must have, I just don't remember what in the world I did!)). While my first job was interesting, rewarding, and challenging, I felt that I had outgrown it and the future prospects didn't really seem all that appealing. I also felt that I didn't know anything. So I went back to school. This time my focus was working towards a PhD in Mathematics.

Part of the trade with working towards a PhD is a graduate stipend. Universities offer a variety of stipends and assistantships. I received a stipend to, in effect, teach. I no longer accurately remember the order in which things happened, but it was something like this: my first course or two was mostly grunt work. Then I got to teach two (? maybe three?) courses per semester. The first few courses were department run courses in Algebra and Pre-calculus. Then eventually, I got to teach the Calculus series. And much like the PLA position, command and respect weren't just handed to me. I didn't have a PhD. I didn't have gray in my beard. I was "just a graduate student". And unlike the PLA position, getting fired meant, in effect, having to drop out of graduate school. So the stakes were a wee bit higher and again it felt that getting fired was easy.

I got lucky. My years as a PLA really helped me to understand how to teach. I enjoyed the courses I taught during graduate school. They were fun courses, I knew how to relate with the students (I was now only seven or eight years older). I loved reading my student comments at the end of the semester. I also learned a lot from what they said. I reflected on my teaching practices, every lecture I gave, every lecture I sat in (I was taking courses at the same time). I learned what good teaching was just as much from taking graduate courses as I did from teaching undergraduate courses. I would discuss teaching woes with the math professors and with my fellow graduate students.

In the first year I taught, my office hours were dead. From my second year on, my office hours were always filled. Students came for help with homework. Sometimes they would come just to talk about their life. Sometimes they would come for advice about what to do with their life, or what courses they should take. I grew as a person, as a mathematician, and as a teacher. I got a teaching award and it felt nice to be recognized.

The changes I made from the first year to the second were "minor" but huge. For starters, I stopped mimicking the stand-and-deliver method that I had become so accustomed to receiving. That's not to say that the stand-and-deliver method is bad or wrong in an absolute sense. It was just that stand-and-deliver was bad for me. I'm a conversationalist. I like engaging in dialogue. I do a far better job at conveying material to students when conversing with them, asking them questions, having them ask me questions, etc. than if I just stood up and filled the chalkboard (yes, we used chalk) with mathematical symbolism.

As soon as I changed my approach from "I speak, you listen. I write, you copy." to "Let's talk math.", I was able to zero in on who was just quiet and brilliant, who was talkative and confused, who was timid and confused, who was timid and brilliant but didn't know he/she was brilliant, etc.

On the first lecture of every class, I had my students make requests for when I should have office hours rather than me telling them when office hours were. And wouldn't you know, students came to office hours for additional help. When it came time for the course mandated exams, I put the date of the exam up for vote. When would it work for everyone? With class sizes of 30 - 40 students, I couldn't satisfy everyone's first choice, but the top two choices pretty much met everyone's scheduling constraints.

The difficulty I had had in my first year was that the advanced students were bored. The struggling students were drowning. And the middle of the road students were always confused. I fixed this by dumping the "traditional" grading scale. The grading scale I used was something like this (with the appropriate "openness" in the interval to avoid grade clash):

0-60	F
60-70	D
70-80	C
80-90	B
90-100	A

The half grades (eg, B-, B+) are tossed in somewhere near the boundary of the letter changes. I realized that I didn't need 60 points to figure out if a student had made no effort. I was "wasting" 60% of the exam. If I really wanted to measure what students knew and really wanted to challenge them, I had to redo this grading system. I really only needed no more than 20 or 30 points to figure out if a student didn't care about the course material. My new grading schema looked like (and assume again, sensible openness on the boundaries and appropriate markings for the half-grades):

0-30	F	You've made little to no effort
30-40	D	You're making an effort, but need to do a bit more
40-60	C	You know your definitions and some basic mechanics
60-85	B	The basics are good and you can combine different concepts
85-100	A	The basics are solid, you can combine different concepts, and you understand how things are interconnected

This allowed me to make real exams. I had been conditioned to believe that a well-written exam would have a class average of 75 out of 100. Upon reflection, this made no sense. With my new grading scheme, I could make an exam where students couldn't simply "memorize" their way into an A. The A students were challenged (though, admittedly even for some, the class was too easy). The B and C students weren't figuring out how to get points by memorizing as much as possible, but were driven to learn the material so that they could actually earn their grade. And the D and F students really could understand how little effort they were making. No student who made a genuine effort to learn failed my course.

I also wanted the grades I gave to mean something *to me* rather than it being a bureaucratic process to follow. I wanted an A to mean mastery of the material (for the course content, of course). I wanted a C to mean, "Ok, you know your definitions, have some of the mechanics understood, but haven't broken out of the formulaic approach, yet.". And I wanted an F to really convey to the student, "you really have made no attempt whatsoever". The grade a student received really opened the door for a conversation. I could really get at the heart of what was going on.

In truth, though, I didn't need the exam to tell me who was going to get what grade. I knew that from lecture — "Let's talk math". I knew it well enough in advance that I would talk to the students directly and express my concerns before the exam (or provide additional challenges if they were sufficiently proficient). Often times they knew they were in trouble as well. And they always had a reason — and I believed all those reasons. I believed it when they told me that they were having trouble adjusting to school, that they were having roommate issues, that they were having health problems, that they had to work two jobs and take 5 courses and it was too much, that they just didn't care, that their parents would only pay for college if the student majored in the sciences, that they partied too much, etc. I felt that the students were honest about their lack of effort.

I hated and still hate failing students. That was / is the worst part of the end of every semester.

Now some may question my connection to the grading system and student motivation. We have to remember that this is college. For 12 years or more, students have been conditioned to hunt for points. By the time they get to me, they already have a standard approach they are going to take towards a class. "What do I have to do to get <grade>?" This is often followed up with questions like "Will there be a curve?" because they are trained to expect the traditional grading scale. This leads to "Will there be extra credit?" because the grading scale is not as efficient as it could be. By dumping the grading scale, it allowed me to make the exams much more meaningful without having to fuss about curves and extra credit. These weren't going to be cream-puff exams and students knew that. They came regularly to office hours, not just the day before the exam. They emailed me with questions. They did their homework. They came to lecture.

By dumping the grading scale, it also threw off the students' standard approach to class. I still got asked those questions, but mostly as an automatic thing to do. It was simply a matter of reminding them of the grading scale. I still did offer extra credit because even this grading scale, even though I felt it was more fair, had some inefficiencies in it. The inefficiency was that the student's final course grade was based on an average over the exams. That was something I couldn't particularly get rid of because of overarching department policies — at least how I understood them. I also felt that I couldn't push the boundaries too much — I was "just a graduate student".

After I finished my PhD, I started working, but I made sure to continue to teach some how, some way. I would adjunct or I would tutor. Before I landed my first post-doctorate job, I adjuncted for one semester at a local college. It was a total disaster. I was teaching an elementary Probability & Statistics course. I thought I had done a reasonably good job of teaching the course, but I apparently didn't teach closely enough to the department-issued final. My students were crushed on the final. When 25 of 25 (or some such ratio) do poorly on the department final, it is hard to argue in defense of the instructor (me). So I won't try to defend it, though I have my defense. Suffice to say, it was unlikely that the college would renew the adjunct contract.

At that point, I had landed a full-time position working as a mathematician in industry. Eventually, I was promoted to head of the math department for the company I worked for — there were 20+ mathematicians that reported to me over probably 8 different time zones. Then things got crazy. It was a demanding and very rewarding job, but I had to put teaching aside since my full-time hours were unreliable — sometimes I'd be in the office until 9pm, come home, eat dinner, and get on the phone with the Australian office. Other times, I'd be up at 5am so I could talk to the Netherlands. Sometimes I had to do both.

I would regularly get phone calls from job seekers asking about what kind of math positions were available. The job seekers were from a variety of mathematics backgrounds. From Bachelor's degrees to PhDs in various areas of mathematics. The questions they asked were universally the same, "You hire people with a math degree?"; "I saw the ad, I thought I only had teaching as an option. How can I get a job in your department?"; "I'm not happy with teaching, what advice do you have for getting a job in industry?". I always took the time out and spoke to them and gave them job search advice. I understood the fog they were in.

I remembered when I was an undergraduate, I chose Electrical Engineering over Mathematics, because Electrical Engineering was "practical" and that "all you could with a Math degree was teach". That's called brainwashing. Because if we stop to think about that statement, we realize that it makes no sense! "I learn Mathematics so I can teach it." That's completely useless! The practicalities of mathematics are lost. Mathematics when part of, say, Electrical Engineering is considered useful. But studying mathematics in the absence of another discipline? That's useless! That view couldn't be more wrong. After having gone through a rigorous mathematics program, I feel free to do *anything*. I will say this, though, in mild contradiction to what I just said: If you want to practice mathematics outside of an academic setting, then learn to program. That's the way the world has gone. Paper and pen solutions need to be implemented into software.

Anyhow, I loved the work, I loved the math department that I ran, but eventually, I had to go it on my own. So, last year I left that job to see what I could do on my own. And this year has been simply amazing. I have loved working with my clients, blogging, meeting other dedicated educators on Twitter, at conferences, at coffee shops, etc. But I've also loved teaching again. I teach as adjunct faculty at a local college and I absolutely love it. Have I said "love" enough?

My industry experience has really helped me to put Mathematics and Mathematics Education in perspective, especially in the context of "why am I learning this?". Now when I look back at my PLA years, my college teaching years, and adjunct teaching prior to working as a mathematician in industry, I can see where I made mistakes, even though, overall, I did what was considered by most standards a great job teaching.

I've also come to realize that there are a lot of ways to teaching mathematics correctly. One of the things I've noticed in discussions with other educators, is that the debate is often focused around multiple ways to do things correctly. While there is value in this, I want to focus on what I consider "Do Not"s. The list below is a non-exhaustive set of "Do Not"s for teaching mathematics. Some of these are mistakes I've made, some are mistakes I've seen my peers make, and some are mistakes I've seen made by professors when I was a student. I use the word "mistake" not in a way to reprimand but more in a way to highlight an approach that could be done better.

Here are a few caveats:

- This is a "do not do" list. As such, it's not a happy-feely thing and it is liable to irritate / anger some folks. My purpose is not to do this. This is not an "attack". The longer a list like this is, the higher the likelihood that someone will be irritated. And the more people that read this ... well, law of large numbers.
- Do not believe everything in this "do not do" list. All of the items listed here are general guidelines. Some can be viewed more rigidly than others, but for the most part it is the context that defines what's good or bad. What works with 18-year old college students won't work with 5-year olds and most certainly will not work with 30+ year-olds. You be the judge.
- Before you reject anything on the list below, ask yourself if you are rejecting it because it truly doesn't apply or because it means it applies to you but you don't want to accept it.
- I'm still guilty, from time to time, of doing some of the things on this list! I amn't perfect.
- When I refer to a group of people, I don't mean every single person. I mean it in the sense of statistically significant portion.

What To Not Do

1. Do not watch your students work. They hate this. It makes them very nervous. They begin to look at your body language to determine if they are doing something correct rather than thinking about if they are doing something correct.
2. Do not use word problems that have no relevance to your students. Five-year olds aren't going to be able to relate / understand a word problem about renting a car, nor will 18-year olds. Leave those types of questions for your older adult students. Five-year olds don't care about / understand money. Fifty-year olds understand money, but they've probably earned as much as they are going to earn and probably don't care either. Money-based examples are good for older teenagers to folks in their mid-forties. Word problems are hard enough for students to translate into mathematical statements, they don't need to also spend time understanding the context. It just adds an unnecessary layer of confusion.

3. Do not laugh at a misconception or an arrived answer that by almost any standard would be "laughable". Ridiculing someone for making a mistake while they are in their learning process goes against everything that it means to be a teacher.

4. Theory without practice is useless. Practice without theory is dangerous. Imagine a surgeon who knows how to perform surgery, but not necessarily where. Or the surgeon who knows in theory how to perform surgery, but never has actually done it.

   - Do not emphasize mechanics only. Mechanics are an essential part of mathematics. Mechanics, however, are not the only part. A student strong in mechanics, but weak in concepts is liable to perform incorrect mathematics with "correct" mechanics.
   - Do not emphasize concepts only. Concepts are good. Philosophy is good. But, understanding the ideas and not being able to execute on them, make the ideas worthless.

5. Do not give answers. Instead, give questions. A typical question from a student is, "What's the next step?". Don't just give the next step, get the student to reason out the process by asking questions like "What is that you want to do?", "What is it that we are trying to solve?", "What methods do we have at our disposal?". It can be painful to the student, so be mindful of this — gradually build up their tolerance to your not giving away answers.

6. Do not harp on your desired use of mathematical symbolism. Instead, focus on the correctness of use rather than adherence to convention.

7. Do not push the symbolism too early. Written mathematics is a mini-language of its own. Imagine if you had to learn mathematics in a language you couldn't read. Would you care to learn mathematics? Today it is very difficult to learn in a formal setting if one is illiterate. Not knowing how to read $\forall\, \varepsilon > 0\ \exists\, \delta > 0 \ldots$ is understandable. Take it slow.

8. Do not always use x as your independent variable and y as your dependent variable. Use *Bob* and *Jack* for x and y or α and β or have students make up their own symbols. Using x and y exclusively or near-exclusively overconditions students to mechanics and hinders their math progress.

9. Do not emphasize "this is how you *have* to solve it". If your goal is to emphasize a certain technique (e.g., integration by parts), then don't fault a student for solving a problem by not using that technique. Instead choose better problems.

10. Do not promote "formula fishing". Students like to ask "What's the formula?" ... giving the formula and having students "plug in" numbers for variables is as much mathematics as retyping a book is reading. It can be a first start, if you like, but eventually we have to get past this.

11. Do not teach students a plethora of 'tricks' and 'gimmicks' for solving specific types of problems. I know where you are coming from if you are not teaching college — you are worried about your students' standardized test scores. All you are doing if you teach tricks and gimmicks is reinforcing the notion that mathematics is a mechanical subject with a myriad of arbitrary rules. I will also go so far as to say that you probably don't know the subject matter as well as you should if that's how you teach mathematics — and that's temporarily ok, just take some time to learn it correctly rather than propagate poor mathematics. Imagine an art teacher who teaches you to draw only by having you follow line by line an already finished work and calls this teaching art. Do the right thing, don't pander to the exam.

12. Do not nitpick syntax too early. Students will write things like $x + 3 = 7 = x = 4$ when solving for x in $x + 3 = 7$. While syntactically this is incorrect, conceptually and mathematically they have solved the problem. They have just written it incorrectly. Once the concepts are straightened out, start nitpicking with the syntax. With that said, you still want to correct bad syntax early, but from an assessment / grading standpoint, err on the side of friendliness rather than militancy.

13. Do not point to the board and say things like "Now we take this and do that with it.". Be very specific, "this" and "that" are unhelpful and do not reinforce vocabulary. It may be tiresome for you, but the more students hear vocabulary, the more they are to use it.

14. Do not emphasize that students memorize definitions and theorems. While having a good recall of mathematical vocabulary can be a distinct advantage for when doing mathematics, learning that vocabulary doesn't have to be via memorization. Memorization is one way, repeated use is another. With that said, do not confuse "Do not emphasize that" with "Never have". A little bit of memorization doesn't hurt, especially about atomic level things.

15. With respect to the previous point, students will often want to refer to a mathematical object or concept as "thing" or some other vague, generic term. This is your chance to get them to use the vocabulary! Do not let this moment pass!

16. Do not answer the question, "Why am I learning this?" with a non-practical answer. If you, the teacher, do not know, then be polite, humble, and respond with, "That's a good question, I don't have a good answer at the moment, but I'll give you an answer tomorrow." Then, tomorrow, give them an answer for where they can use what they are learning. Despite the question of "why", the student really is asking "Where am I going to use this?". Also, do not give tautological answers like "So you can take the next subject." or "So you can teach it.".

    - For non-graduate level work, give a proper, applied example — think physics, chemistry, biology, computer science, programming, finance, any of the engineering disciplines, starting a business, grocery shopping, retail work, crocheting, practicing law, drawing, designing, decorating, playing games, sports, etc. Every one of these areas *can* use mathematics (some more so than others and with varying levels of sophistication) if the person knows mathematics and there are advantages that can be gained by having this knowledge.
    - For graduate-level work, give a research level response that's not "So you can take Advanced Stochastic Calculus". Often times, any one topic can't be used in isolation in practice. This is the mistake in reasoning many people fall into when wondering about the practicalities of mathematics.
    - Do not give oversimplified and contrived answers for where math is used. Doing so just opens up another round of inane debate where the non-mathematician will counter with "I can do that without math.". I continue to make this mistake, though less often now than previously. The simple truth is, they are likely not doing those things without mathematics. They are, in all probability, using basic mathematical concepts, but they don't consider that to be mathematics. Either that or they are extremely lucky.

17. Do not extrapolate your lack of use of certain topics in mathematics in your life to conclude that everyone does not use that topic. In other words, do everything you can to resist turning your inabilities into your students' inabilities. I've been witness to math *professors* doing this.

18. I know you have a class to teach and you have to get through material by a certain amount of time. But you teaching it and the students learning it are two different things. If your students aren't learning the material, find another way to teach it. It's not a guaranteed solution, but continuing to teach material in a way that your students don't understand is craziness on your part. It's a tricky balance, I am aware of this — it's hard to reach every student in your classroom of 25 or more students and at some point (large lecture halls, e.g.) you only have so much variety in pedagogy that's available. But use whatever variety you have at your disposal. Break the "rules of teaching" (without breaking the law) that you've been taught.

19. Do not grade homework, worksheets, quizzes, in-class assignments, attendance, or any other such "assignment" that should remain formative. If you are going to grade anything it ought to be an

exam. Micrograding, especially when it influences the final course grade, does not give an accurate reflection of student learning. What difference does it make if a student scored poorly on quiz 3, homework assignment 2, and exam 1 if by the end of the course the student knows the course material to an "A" level? Remember, it's a classroom for learning mathematics not for synchronized swimming. There are some exceptions to this, but they are few and far between.

20. Do not give deadlines for every thing. I understand the counterargument, "If I don't give a deadline, I'll just get everything at the end of the semester." I've dropped the deadline for some time now, and my experience has been quite the opposite. Students turn work in when they are ready to do so. I just remind them about assignments not yet received. I won't deny that there is a bit of an uptick at the end of the semester, but I'll pay that price if it means that students are learning more on their pace than on mine / the class dictated pace.

21. Do not teach via compliance. Dropping the micrograding and dropping deadlines takes away the bureaucratic pressure for learning. By getting rid of these types of pressures, the responsibility of learning falls on the student. Learning gets far closer to student choice than the veiled choice that we say students have with the one-way game of "if you don't do it, then you'll get a zero.". (For those wondering, I do unfortunately have to engage in some of the micrograding that I say not to do. But this is when I have "department-run" courses for which the syllabus and grading criteria are out of my hands. For these courses, I minimize the impact of the micrograding by selecting the lowest possible allocation for non-exam related items.)

22. Do not believe that "worksheets are evil". Worksheets aren't evil. Worksheets are worksheets. It's what you do with worksheets that can be evil. If you use them to inflict thousands of papercuts on people or animals, that's evil. If you use them as a substitute for conceptual understanding that's bad teaching. If you use them as reinforcement of mechanics that's not so bad. If you use them as an optional form of exercise for students whose mechanics need work, that's pretty good.

23. Do not believe that "homework is evil". Homework isn't evil. Homework is homework. It's what you do with homework that can be evil. Just keep in mind that assigning homework and making it mandatory is exerting control over your student's life outside of your classroom. Ask yourself if that makes sense. Then remind yourself that your students have more than your course. If you want to assign homework, then make it optional, ungraded, and without a deadline.

24. Do not believe that "extra credit is evil". Ask yourself if the grade you assign to your students at the end of the course is an accurate reflection of what they have learned in aggregate. An average does not do this — think about it, if a student understands the material to an "A" level one week after exam 1 but received a "C" on exam 1, why are they being penalized for learning something "late"? That C is stale information. Sometimes you don't have a choice in the grading schema because it is dictated by an external authority. Use extra credit to fix this inequity. It's a bit ugly and hackish, but it's better than penalizing the student who doesn't learn at the pace dictated by the course. It's not synchronized swimming! Find a way to fix as many inequities as possible in your grading system.

25. With respect to extra credit, do not absolve students of their responsibility to learn. But ... see the next "Do not".

26. Do not think that grading assignments teaches responsibility. You can't mandate responsibility. Responsibility is something to discuss with someone when they have a choice. Give your students the choice to learn without the albatross of grading bureaucracy looming over them. See what happens. You may be positively surprised.

27. Do not teach / assess solely from a checklist. Mathematics is not to be learned via a checklist. I use the word checklist in place of syllabus. It is fine to have standards that have to be mastered. It is good quality control for the teacher to make sure that the teacher is covering the requisite topics. But

teaching individual ideas, concepts, techniques and never putting them together is depriving students of mathematics. For example, a good calculus student may know the standard integration techniques as individual techniques, but a calculus student who really understands the techniques should be able to integrate, for example,

$$\int \sqrt{x^2+1}\,dx$$

28. Math has to marinate. Do not rush learning. Provide repeated exposure from as many angles as possible and give it some time.

29. Do not propagate the notion "either you get it or you don't". I have heard too many math teachers and professors say this. Mathematics is not like that! Very few subjects are! If the student isn't "getting it", try to figure out why. Sometimes they don't care and that it is what it is. Other times, it's you who doesn't care. Still other times, it's a clash between how you understand it and how the student can understand it. Teaching mathematics isn't easy.